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OPERATIONAL MODAL ANALISYS USING JOINT STATISTICAL ANALYSIS OF MULTIPLE RECORDS

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ABSTRACT

In Operational Modal Analysis (OMA) of a structure, the data acquisition process may be repeated many times. In these cases, the analyst has several similar records for the modal analysis of the structure that have been obtained at different time instants (multiple records). The solution obtained varies from one record to another, sometimes considerably. The differences are due to several reasons: statistical errors of estimation, changes in the external forces (unmeasured forces) that modify the output spectra, appearance of spurious modes, etc. Combining the results of the different individual analysis is not straightforward. To solve the problem, we propose to make the joint estimation of the parameters using all the records. This can be done in a very simple way using state space models and computing the estimates by maximum-likelihood. The method provides a single result for the modal parameters that combines optimally all the records.

1. INTRODUCTION

The modal parameters of a structure are obtained from the measured vibrations in a two-step procedure: in the first step, a parametric model of the structure is estimated from the data; in the second step, the modal parameters are calculated from the identified model.

The state space model is the most used parametric model. This model emerge naturally from the differential equation of motion of the structure, and its formulation is related to physical parameters like mass, stiffness and damping, and therefore, to the modal parameters.

Several methods exist in the literature to estimate a state space model from measured vibration data [1, 2]. Probably, the most popular one is the Stochastic Subspace Identification method (SSI). A complete overview of SSI is provided in [3].

The data acquisition process may be repeated many times, so the analyst has several similar records for the modal analysis of the structure that have been obtained in different experiments. The traditional way to address this situation with the subspace method consists on performing a separate analysis for each experiment, and then averaging or combining the results. But the solutions obtained vary from one record to another, sometimes considerably, and matching modes coming from different data is not simple, due to: (1) not all modes appear in all records and, (2) the statistical errors inherent to the estimation provide different results for the same mode in different records, and these errors are especially important in the damping. The final solution should be combination of the individual solutions. The most important modes of the structure tend to appear in the majority of the records, while the spurious and/or weakly excited ones are only detected in individual records. The criteria for discarding modes and the method for combining the different true estimates are not clear and are almost always subjective decisions requiring an expert opinion.

In this paper we propose a complete methodology for the joint estimation of the model parameters using information from multiple records. It is based on the Expectation-Maximitation (EM) algorithm [4, 5] that provides the maximum-likelihood estimators of the parameters. The joint estimation of the structure's parameters combines information optimally: the modes that are repeated in different records are detected more clearly while modes specific to some records tend to blur in the joint analysis [6]. The method is finally extended to take into account both the "common" modes and the "specific" modes as well.

The method is applied to a real example, the Tablate II Bridge, located on a road south of Granada (Spain). The results presented illustrate the potential and usefulness of the procedure.

2. STATE SPACE MODELS FOR MULTIPLE RECORD ANALYSIS

Consider a linear, time invariant mechanical system (bridges or buildings in the context of civil engineering). We put n_o sensors in the system and measure M different records for the system output (acceleration in this work). The data can be represented by

$$Y_N^{(r)} = \{y_1^{(r)}, y_2^{(r)}, \dots, y_N^{(r)}\}, \quad r = 1, 2, \dots, M \tag{1}$$

where $y_t^{(r)} \in \mathbb{R}^{n_o}$ is the measured output vector for record r and at time instant t. For simplicity, we consider all the records have the same length, N. Therefore, each record has $n_o \times N$ data (n_o channels with N data each), and in total, we have $n_o \times N \times M$ data. It is important to remark that the records are obtained at different moments (so they are not simultaneously measured data) and the sensors do not change their position from one record to the next.

The objective is to estimate the modal parameters of the system using the M records. As stated in the Introduction, the modal parameters are extracted from the measured vibrations in a two-step procedure:

- Step 1: a parametric model of the structure is estimated from the data. We are going to restrict our models to state space models. For OMA analysis, a stochastic state space model is generally used

$$x_{t+1} = Ax_t + w_t, \quad w_t \rightsquigarrow N(0, Q) \tag{2a}$$

$$y_t = Cx_t + v_t, \quad v_t \rightsquigarrow N(0, R) \tag{2b}$$

  where t denotes the time instant, of a total number N, measured with constant sampling time Δt; $y_t \in \mathbb{R}^{n_o}$ is the measured output vector; $x_t \in \mathbb{R}^{n_s}$ is the state vector; n_o and n_s are the number of outputs and the order of the state vector, respectively; $A \in \mathbb{R}^{n_s \times n_s}$ is the transition state matrix describing the dynamics of the system; $C \in \mathbb{R}^{n_o \times n_s}$ is the output matrix, which is describing how the internal state is transferred to the the output measurements y_t. The noise vectors, $w_t \in \mathbb{R}^{n_s}$ and $v_t \in \mathbb{R}^{n_o}$, comprise unmeasurable signals. They are assumed to be zero-mean, white noise sequences with covariance matrices Q and R, respectively. For simplicity, we also consider that they are independent.

- Step 2: the modal parameters are calculated from the identified model, in particular, from matrices A and C. Assuming viscous damping, it is customary to express the eigenvalues of matrix A by (see [8]):

$$\lambda_j = \exp\left[\left(-\zeta_j\omega_j \pm i\omega_j\sqrt{1-\zeta_j^2}\right)\Delta t\right], \tag{3}$$

where ω_j is the natural frequency, ζ_j is damping ratio, and Δt is the time step (the eigenvalues of A come in complex conjugate pairs and each pair represents one physical vibration mode). Therefore

$$\omega_j = \frac{\left|\ln(\lambda_j)\right|}{\Delta t}, \tag{4}$$

$$\zeta_j = \frac{-\mathrm{Real}\left[\ln(\lambda_j)\right]}{\omega_j\Delta t}. \tag{5}$$

The jth mode shape $\phi_j \in \mathbb{C}^{n_o}$ evaluated at sensor locations can be obtained using the following expression:

$$\phi_j = C\psi_j, \tag{6}$$

where ψ_j is the complex eigenvector of A corresponding to the eigenvalue λ_j.

Here we propose three different state space models that can be used to analyze multiple records.

2.1 State space model 1

The first approach is the most direct and naive one, that is, to estimate the well known state space model (2)

$$\begin{aligned} x_{t+1} &= Ax_t + w_t, \quad w_t \rightsquigarrow N(0,Q)\\ y_t &= Cx_t + v_t, \quad v_t \rightsquigarrow N(0,R) \end{aligned}$$

for each record r. Because of there are M different records, the process can be expressed as

$$x_{t+1}^{(r)} = A^{(r)}x_t^{(r)} + w_t^{(r)}, \quad w_t^{(r)} \rightsquigarrow N(0,Q^{(r)}) \tag{8a}$$

$$y_t^{(r)} = C^{(r)}x_t^{(r)} + v_t^{(r)}, \quad v_t^{(r)} \rightsquigarrow N(0,R^{(r)}) \tag{8b}$$

$r = 1, 2, \ldots, M$. Therefore, the unknown parameters are

$$\theta^{(r)} = \{A^{(r)}, C^{(r)}, Q^{(r)}, R^{(r)}, \mu_0^{(r)}, \Sigma_0^{(r)}\}, \quad r = 1, 2, \ldots, M \tag{9}$$

where $\mu_0^{(r)}$ and $\Sigma_0^{(r)}$ are the mean and variance of the initial state $x_0^{(r)}$ respectively (which is assumed to be normal distributed). Since the modal parameters are computed from matrices $A^{(r)}$ and $C^{(r)}$, and since there are M different estimates, M different values for the modal parameters are obtained:

$$A^{(r)}, C^{(r)} \Rightarrow \omega_j^{(r)}, \zeta_j^{(r)}, \phi_j^{(r)}, \quad j = 1, 2, \ldots, n_s/2, \quad r = 1, 2, \ldots, M.$$

The most important modes of the structure should appear in most of the records. Then, individual solutions can be combined to obtain a single estimation of each mode: modal parameters that seem to represent the same physical mode are grouped, and mean values can be computed at the end. However, mode pairing between setups is difficult due to closely spaced modes, different excitation levels, spurious modes,... The following state space model is designed to overcome these drawbacks.

2.2 State space model 2

The state space model we propose to use for multiple records is

$$x_{t+1}^{(r)} = Ax_t^{(r)} + w_t^{(r)}, \quad w_t^{(r)} \rightsquigarrow N(0, Q^{(r)}) \tag{10a}$$

$$y_t^{(r)} = Cx_t^{(r)} + v_t^{(r)}, \quad v_t^{(r)} \rightsquigarrow N(0, R^{(r)}). \tag{10b}$$

It is important to note that matrix A is the same for all the records because the system is time invariant, and matrix C is also constant because the sensors location do not change. On the other hand, $x_t^{(r)}$, $w_t^{(r)}$ and $v_t^{(r)}$ are record-depending because the system outputs $y_t^{(r)}$ are different from record to record.

The unknown parameters of this model are

$$\theta = \{A, C, Q^{(r)}, R^{(r)}, \mu_0^{(r)}, \Sigma_0^{(r)}\}, \quad r = 1, 2, \ldots, M \tag{11}$$

The result of this model is a single estimation for matrices A and C, so a single value for each modal parameter is obtained

$$A, C \Rightarrow \omega_j, \zeta_j, \phi_j, , \quad j = 1, 2, \ldots, n_s/2.$$

So paring modes between setups is not needed in this case. And, what is more important, matrices A and C (and therefore the modal parameters) are estimated using all the available information.

2.3 State space model 3

Model (10) assumes that the same modal parameters are present in all the records. However, in practice this may not be true because the input or excitation forces are non-white and different from record to record, so they may excite different modal parameters in different records. Besides, non-white excitations can introduce poles in matrix A which are indistinguishable from poles corresponding to physical modes because the inputs are not measured; since the excitations vary from record to record, these poles vary as well. Model (10) tends to discard these record-depending or specific modal parameters and only estimates common modal parameters, the parameters that are common to all the records. But the specific modal parameters can be important in a given record, and discarding them can affect to the estimation of the common parameters; or maybe we can be interested in some modal parameters that are present only in certain records. For these reasons we propose a state space model that allow to estimate the common modal parameters and the specific modal parameters as well. The model is

$$\begin{bmatrix} x_{t+1}^{(1,r)} \\ x_{t+1}^{(2,r)} \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2^{(r)} \end{bmatrix} \begin{bmatrix} x_t^{(1,r)} \\ x_t^{(2,r)} \end{bmatrix} + \begin{bmatrix} w_t^{(1,r)} \\ w_t^{(2,r)} \end{bmatrix}, \quad \begin{bmatrix} w_t^{(1,r)} \\ w_t^{(2,r)} \end{bmatrix} \rightsquigarrow N\left(0, \begin{bmatrix} Q_{11}^{(r)} & Q_{12}^{(r)} \\ Q_{21}^{(r)} & Q_{22}^{(r)} \end{bmatrix}\right) \tag{12a}$$

$$y_t^{(r)} = \begin{bmatrix} C_1 & C_2^{(r)} \end{bmatrix} \begin{bmatrix} x_t^{(1,r)} \\ x_t^{(2,r)} \end{bmatrix} + v_t^{(r)}, \quad v_t^{(r)} \rightsquigarrow N(0, R^{(r)}) \tag{12b}$$

where $A_1 \in \mathbb{R}^{n_{s1} \times n_{s1}}$ contains the eigenvalues common to all the records, and $A_2^{(r)} \in \mathbb{R}^{n_{s2} \times n_{s2}}$ contains the eigenvalues specific to the record r. Therefore, the matrix C has also two components: C_1 for the common eigenvalues, and $C_2^{(r)}$ for the eigenvalues specific to the record r.

The unknown parameters of this model are now

$$\theta = \{A_1, A_2^{(r)}, C_1, C_2^{(r)}, Q_{11}^{(r)}, Q_{12}^{(r)}, Q_{22}^{(r)}, R^{(r)}, \mu_0^{(r)}, \Sigma_0^{(r)}\}, \quad r = 1, 2, \ldots, M \tag{13}$$

The modal parameters are now

- Common modal parameters

$$A_1, C_1 \Rightarrow \omega_j^{(1)}, \zeta_j^{(1)}, \phi_j^{(1)},, \quad j = 1, 2, \ldots, n_{s1}/2.$$

- Specific modal parameters

$$A_2^{(r)}, C_2^{(r)} \Rightarrow \omega_j^{(2,r)}, \zeta_j^{(2,r)}, \phi_j^{(2,r)}, \quad j = 1, 2, \ldots, n_{s2}/2, \quad r = 1, 2, \ldots, M.$$

3. MAXIMUM LIKELIHOOD ESTIMATION: THE EM ALGORITHM

To estimate the state space models (8), (10) and (12), we propose to use Maximum Likelihood Estimation and the Expectation Maximization algorithm. The method is described in [4], and applications to OMA analysis can be found in [5] and [6].

3.1 Estimation of model 1

We estimate M different state space models, one for each record. These models can be estimated using any of the methods used in the literature (see [2] for different methods). The Stochastic Subspace Identification method is probably the most used one. We are going to use the Expectation-Maximization algorithm because is the method we propose to use to estimate model 2 and model 3. The method is applied as indicated in [5].

3.2 Estimation of model 2

First we need to compute the likelihood: consider we know the observed values $Y_N^{(r)} = \{y_1^{(r)}, y_2^{(r)}, \ldots, y_N^{(r)}\}$ and also the states $X_N^{(r)} = \{x_1^{(r)}, x_2^{(r)}, \ldots, x_N^{(r)}\}$. The joint density function for record r is given by (see [6])

$$f_{\theta^{(r)}}(X_N^{(r)}, Y_N^{(r)}) = f_{\mu_0^{(r)}, \Sigma_0^{(r)}}(x_0^{(r)}) \prod_{t=1}^{N} f_{A,Q^{(r)}}(X_t^{(r)} | X_{t-1}^{(r)}) \prod_{t=1}^{N} f_{C,R^{(r)}}(Y_t^{(r)} | X_t^{(r)}). \tag{14}$$

where under Gaussian assumption

$$f_{\mu_0^{(r)}, \Sigma_0^{(r)}}(x_0^{(r)}) = \frac{1}{(2\pi)^{n_s/2} |\Sigma_0^{(r)}|^{1/2}} \exp\left(-\frac{1}{2}\left(x_0^{(r)} - \mu_0^{(r)}\right)^T \left(\Sigma_0^{(r)}\right)^{-1} \left(x_0^{(r)} - \mu_0^{(r)}\right)\right),$$

$$f_{A,Q^{(r)}}(X_t^{(r)} | X_{t-1}^{(r)}) = \frac{1}{(2\pi)^{n_s/2} |Q^{(r)}|^{1/2}} \exp\left(-\frac{1}{2}\left(x_t^{(r)} - A x_{t-1}^{(r)}\right)^T \left(Q^{(r)}\right)^{-1} \left(x_t^{(r)} - A x_{t-1}^{(r)}\right)\right),$$

$$f_{C,R^{(r)}}(Y_t^{(r)} | X_t^{(r)}) = \frac{1}{(2\pi)^{n_o/2} |R^{(r)}|^{1/2}} \exp\left(-\frac{1}{2}\left(y_t^{(r)} - C x_t^{(r)}\right)^T \left(R^{(r)}\right)^{-1} \left(y_t^{(r)} - C x_t^{(r)}\right)\right),$$

Thus, if we consider M independent registers, the joint density function $f_\theta(X_N, Y_N)$ will be the product of individual ones

$$f_\theta(X_N, Y_N) = \prod_{r=1}^{M} f_{\theta^{(r)}}(X_N^{(r)}, Y_N^{(r)}) \tag{15}$$

This is known as the complete data likelihood, $L_{X_N, Y_N}(\theta) = f_\theta(X_N, Y_N)$. In practice we generally work with the log-likelihood, so information is combined by addition and it can be written as a sum of the log-likelihood of each individual record:

$$\log L_{X_N, Y_N}(\theta) = \sum_{r=1}^{M} \log f_{\theta^{(r)}}(X_N^{(r)}, Y_N^{(r)}). \tag{16}$$

The maximum likelihood method consists on maximizing this equation. However, this expression cannot be computed because the states are unknown. We propose to use the Expectation Maximization algorithm to solve this situation. It consists on two steps:

- Expectation step (E-step): since the states are unknown, we compute the expected value of Equation (16). The available data are the observations and we also need an initial value for the parameters θ_k. Thus, we compute $\mathrm{E}\left[\log L_{X_N,Y_N}(\theta)|Y_N^{(r)},\theta_k\right]$. This conditional expectation is computed using the Kalman filter outputs.

- Maximization step (M-step): the maximum of $\mathrm{E}\left[\log L_{X_N,Y_N}(\theta)|Y_N^{(r)},\theta_k\right]$ with respect to the parameters θ will yield the update estimate θ_{k+1}. This is the strong point of the EM algorithm because the maximum values are obtained from explicit formulas (see [6]).

The two steps, E and M-steps, are repeated iteratively until the maximum is reached.

3.3 Estimation of model 3

Model (12) can be expressed in a more compact form by

$$x_{t+1}^{(r)} = A^{(r)}x_t^{(r)} + w_t^{(r)}, \quad w_t^{(r)} \rightsquigarrow N(0,Q^{(r)}) \tag{17a}$$
$$y_t^{(r)} = C^{(r)}x_t^{(r)} + v_t^{(r)}, \quad v_t^{(r)} \rightsquigarrow N(0,R^{(r)}) \tag{17b}$$

where

$$A^{(r)} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2^{(r)} \end{bmatrix}, \qquad C^{(r)} = \begin{bmatrix} C_1 & C_2^{(r)} \end{bmatrix}. \tag{18}$$

Let be again the observed values $Y_N^{(r)} = \{y_1^{(r)}, y_2^{(r)}, \ldots, y_N^{(r)}\}$ and the states $X_N^{(r)} = \{x_1^{(r)}, x_2^{(r)}, \ldots, x_N^{(r)}\}$. The joint density function for record r is given now by

$$f_{\theta^{(r)}}(X_N^{(r)}, Y_N^{(r)}) = f_{\mu_0^{(r)},\Sigma_0^{(r)}}(x_0^{(r)}) \prod_{t=1}^{N} f_{A^{(r)},Q^{(r)}}(X_t^{(r)}|X_{t-1}^{(r)}) \prod_{t=1}^{N} f_{C^{(r)},R^{(r)}}(Y_t^{(r)}|X_t^{(r)}). \tag{19}$$

where under Gaussian assumption

$$f_{\mu_0^{(r)},\Sigma_0^{(r)}}(x_0^{(r)}) = \frac{1}{(2\pi)^{n_s/2}|\Sigma_0^{(r)}|^{1/2}} \exp\left(-\frac{1}{2}\left(x_0^{(r)} - \mu_0^{(r)}\right)^T \left(\Sigma_0^{(r)}\right)^{-1} \left(x_0^{(r)} - \mu_0^{(r)}\right)\right),$$

$$f_{A^{(r)},Q^{(r)}}(X_t^{(r)}|X_{t-1}^{(r)}) = \frac{1}{(2\pi)^{n_s/2}|Q^{(r)}|^{1/2}} \exp\left(-\frac{1}{2}\left(x_t^{(r)} - A^{(r)}x_{t-1}^{(r)}\right)^T \left(Q^{(r)}\right)^{-1} \left(x_t^{(r)} - A^{(r)}x_{t-1}^{(r)}\right)\right),$$

$$f_{C^{(r)},R^{(r)}}(Y_t^{(r)}|X_t^{(r)}) = \frac{1}{(2\pi)^{n_o/2}|R^{(r)}|^{1/2}} \exp\left(-\frac{1}{2}\left(y_t^{(r)} - C^{(r)}x_t^{(r)}\right)^T \left(R^{(r)}\right)^{-1} \left(y_t^{(r)} - C^{(r)}x_t^{(r)}\right)\right),$$

For M independent registers, the complete data likelihood will be

$$L_{X_N,Y_N}(\theta) = \prod_{r=1}^{M} f_{\theta^{(r)}}(X_N^{(r)}, Y_N^{(r)}) \tag{20}$$

And taking logarithms

$$\log L_{X_N,Y_N}(\theta) = \sum_{r=1}^{M} \log f_{\theta^{(r)}}(X_N^{(r)}, Y_N^{(r)}). \tag{21}$$

The maximum of this equation can be computed using the EM algorithm. This algorithm is applied in the same way as in Section 3.2 but now taking into account the special form of $A^{(r)}$ and $C^{(r)}$ given by Equation (18).

3.4 Comments about the estimation algorithm

We propose to estimate the three state space models using the EM algorithm. We want to make some comments about this method:

- Models 2 and 3 perform a joint estimation of the modal parameters. We have indicated in the Abstract and in the Introduction that the parameters estimated using these models combine optimally the information. With optimally we mean that since we are using maximum likelihood, the joint analysis gives more weight to those parameters that are more likelihood taking into account all the records.

- The EM algorithm is iterative, and therefore it needs a starting point. Depending on the starting point, a local maximum can be reached instead of the global maximum (see [5]). To avoid this problem, we use a starting point that is very close to the global maximum: we apply the Stochastic Subspace Identification (SSI) method and use the obtained result as a starting point for the EM (in the case of Models 2 and 3 we apply SSI to one of the available record).

4. EXAMPLE: TABLATE II BRIDGE, GRANADA (SPAIN)

Figure 1: The Tablate II Bridge under construction (similar to the Tablate I Bridge in service).

	Record name	Date (yy-mm-dd)	Shot channel	Temperature (°C)
Record 1	DN187	2009-04-11	Vertical	24.5
Record 2	DN243	2009-04-19	Transversal	24.5
Record 3	DN313	2009-04-24	Transversal	24.5
Record 4	DN329	2009-04-25	Longitudinal	24.5

Table 1: Multiple records from Tablate Bridge.

Figure 1 shows the construction of the Tablate II Bridge, located on a highway in the south of Granada (Spain). The structure of the bridge consists of a concrete deck resting on a metal arch with a span of 128 meters. The bridge has 3 accelerometers permanently installed at the center of the arch, on the sidewalk. The system automatically saves the recorded accelerations of the bridge when any of the three signals exceeds a certain threshold. The data were sampled at a rate of 250 Hz. A total of 10000 samples (40 seconds) was acquired for each channel and each record. The ambient excitation sources of the bridge were wind

State space Model 1								State space Model 2		State space Model 3	
Record 1		Record 2		Record 3		Record 4					
f (Hz)	ζ (%)	f (Hz)	ζ (%)	f (Hz)	ζ (%)	f (Hz)	ζ (%)	f (Hz)	ζ (%)	f (Hz)	ζ (%)
1,058	1,57	1,053	1,02	1,048	0,32	1,045	0,24	1,050	0,62	1,046	0,43
1,116	0,83	1,092	1,67	1,108	1,98	1,095	0,36	1,101	0,90	1,092	0,63
1,637	1,28	1,677	0,62	1,688	0,85	1,634	1,79	1,672	1,17	1,675	0,96
2,792	2,01	2,779	0,87	2,740	0,34	1,896	6,40	2,801	1,19	2,734	1,76
2,902	0,35	2,923	0,86	2,755	3,11	2,821	0,78	2,874	9,33	2,847	1,38
3,010	8,02	3,589	1,26	2,912	0,16	2,920	2,38	2,903	0,34	2,913	0,46
3,574	0,55	4,058	7,45	3,736	4,13	3,021	33,47	3,588	1,13	3,581	0,86
4,093	3,26	4,323	2,48	3,900	11,21	4,161	7,48	4,166	3,15	4,111	4,75
4,435	0,44	4,844	1,25	4,455	2,18	4,934	1,63	4,427	1,90	4,455	0,84
4,755	2,35	5,212	2,55	4,869	2,53	5,487	0,34	4,883	1,60	4,878	1,64

Table 2: Frequencies and modal damping for the selected records. The results provided by the EM algorithm (n_s=20) using the three proposed models are displayed.

and traffic mainly. These accelerometers were installed to monitor possible changes in the structure of the bridge over time. Such a small number of sensors does not allow the modal shapes to be conveniently estimated, so the analysis will address only the estimation of the modal frequencies and the damping ratios.

The described recording system produces about 20 different records a day. In order to illustrate the methodology described in the paper, we have chosen four different records obtained on April, 2009. Many researchers have reported the influence of temperature in modal parameters (for instance [7]). For this reason, we have chosen data recorded at the same temperature. We have also chosen data from different days to be sure the ambient loads are totally different. Table 1 summarizes the characteristics of the records we have used.

Table 2 shows the results obtained with the proposed models. We have estimated ten modes (the order of the state space models is n_s = 20). For Model 1, one can see that the frequencies and damping ratios estimated using record 1 are different to the ones estimated using records 2, 3 and 4. Therefore, it is complex to decide if similar estimates correspond to the same physical mode. Six of the ten modes seem to appear in the four records: around 1.05 Hz, 1.10 Hz, 1.60 Hz, 2.80 Hz, 2.90 Hz, 4.90 Hz. Other estimates are present in three records: around 3.6 Hz, 4.1 Hz, and 4.4 Hz, and the rest are present in one or two records. In any case, this pairing process is far from being easy.

The results obtained using Model 2 are included in the following two columns. Frequencies obtained with this model (the joint analysis) correspond to the values that are most repeated in the individual analysis. All modal frequencies appearing in three or more individual records also appear in the joint estimation. In general, when a frequency appears only in a single record, it does not appear in the joint estimation.

In the analysis of individual records (Model 1), the damping values present higher variability than the frequencies. Some values are higher than 5%, and could be considered over acceptable limits. However, in the joint estimate, damping values are lower and within the range of admissibility (except mode with frequency 2.874 Hz and damping ratio 9.33%). Another detail that catches the eye is that the overall damping value is significantly lower than the average values of the individual ones.

The results from Model 3 (also a joint analysis) are included in the last columns. We have considered ten common modes and one specific. The estimates are very similar to the ones obtained with Model 2. The only differences appear in the frequency range [2.50 - 3.00] Hz. It is remarkable that the mode estimated with high damping ratio in Model 2 (2.874 Hz - damping ratio 9.33%) is not present in Model 3.

The results of Table 2 are included in Figure 2 as well. Blue arrows indicate frequencies estimated using Model 1; red arrows stand for: Figure (a) - frequencies estimated using Model 2, Figure (b) - common frequencies estimated using Model 3. The green arrows indicate specific frequencies in Model 3.

From the above analysis we see that joint estimation results do not match the individual mean values, neither for the frequencies nor for the damping. The joint analysis presented here combines information efficiently, giving more weight to those estimates that are more likely. The joint estimation of a mode includes the information from all records, even those that have not obtained the mode in question in their individual analysis.

Another important aspect is the whether an estimated parameter corresponds to a physical mode of vibration or, on the contrary, is an spurious or mathematical one. The most used method is to build the so-called stabilization diagram: it consists on to estimate the state space model for a wide range of orders and the eigenfrequencies obtained for all these orders are plotted in an eigenfrequency vs. model order diagram [9]. Experience on a very large range of problems shows that in such analysis, the eigenfrequencies corresponding to physical modes appear at most of the used model orders, while mathematical and spurious poles tend to scatter around the frequency range.

Figure 3 shows the stabilization diagram built using Model 2. At n_s=20 (the order used in Table 2) we see that modes 5 and 8 are not stable. In fact, they are the modes with high damping ratio. Mode 10 must be taken with care because it shows an erratic behaviour in the stabilization diagram. The rest of the modes are stable for almost all the orders shown in the diagram, what gives us confidence about they correspond to physical modes of the bridge.

5. CONCLUSIONS

The data acquisition process may be repeated many times in OMA, so the analyst has several similar records for the modal analysis of the structure that have been obtained at different time instants. The solutions obtained vary from one record to another, sometimes considerably. The final solution should be an averaging of the individual solutions.

In this paper we propose a complete methodology for the joint estimation of the model parameters. The joint estimation of the structure's parameters combines information optimally: the main modes that are repeated in different records are detected more clearly while modes specific to some records tend to blur in the joint analysis. The method can be extended to take into account the "common" modes and the "specific" modes as well.

Joint estimation results do not match the individual mean values, neither for the frequencies nor for the damping. The joint analysis presented here combines information more efficiently, giving more weight to those estimates that are more likely. It is important to note that the joint estimation of a mode includes the information from all records, even those that have not obtained the mode in question in their individual analysis.

In conclusion, estimating parameters using all the recorded information and the EM algorithm, has numerous advantages in the modal analysis of a structure, and resolves the difficulties in combining the individual solutions coming from different records.

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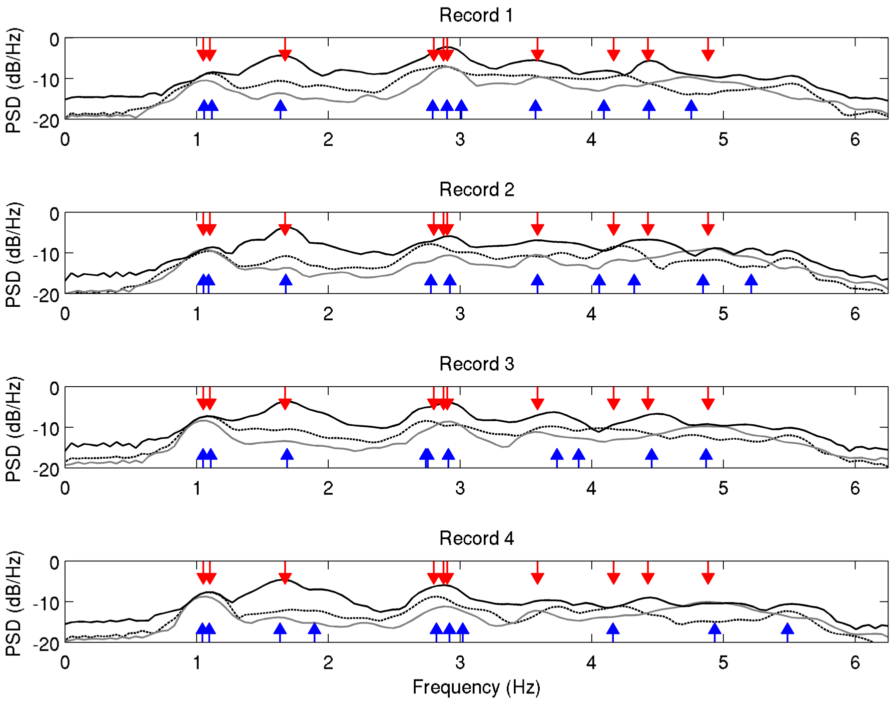
(a)



(b)

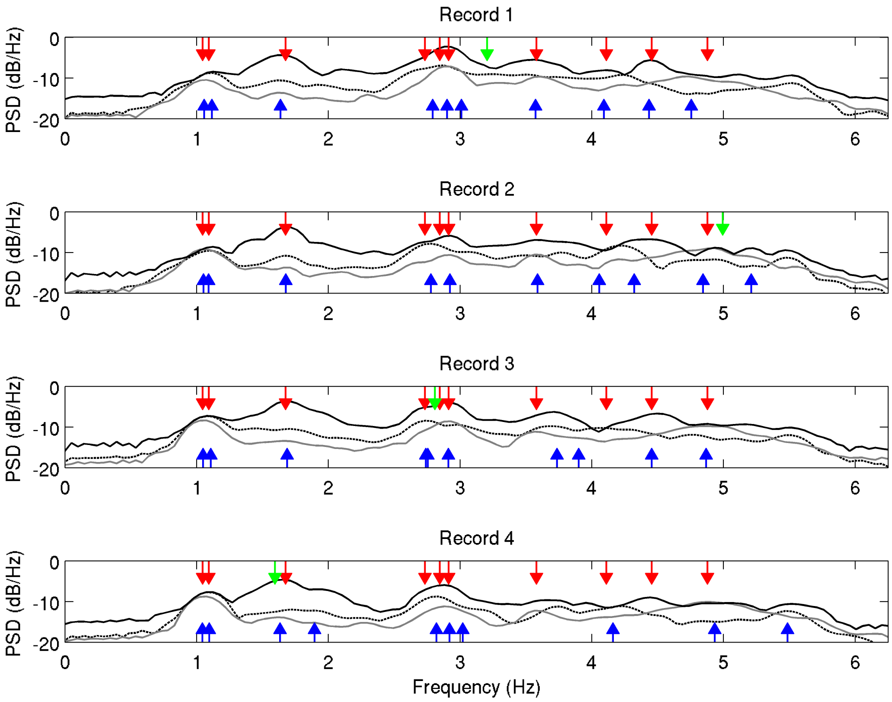


Figure 2: Spectral density functions (estimated by Welch method) of signals measured at the bridge (black - vertical direction, grey - longitudinal direction, dotted - transversal direction). Blue arrows indicate frequencies estimated using Model 1 with each record; red arrows stand for: Figure (a) - frequencies estimated using Model 2, Figure (b) - common frequencies estimated using Model 3. The green arrows indicate specific frequencies in Model 3.

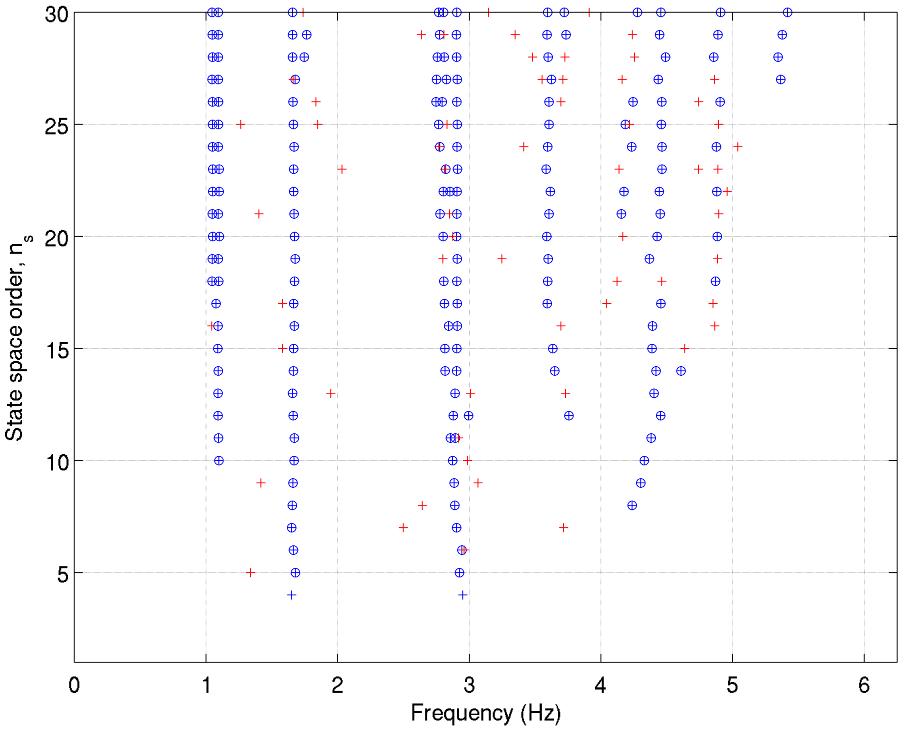


Figure 3: Stabilization diagram computed by Method 2. The criteria are: 2% for frequencies, 5% for damping ratios, 5% for mode shape vectors (MAC). $\oplus$: stable mode, $+$: unstable mode.

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